

Derivation of Count per Milliliter from Percentage of Solids

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Introduction

Our strength is in offering you a complete particle technology. We give you simple protocols for working with particles. We provide you concrete data, backed by years of applications research in our own labs. The following procedure outlines the suggested process for calculating Thermo Scientific polymer particles from percentage of solids to the number of particles per milliliter of suspension.

Procedure

Following is the derivation of the number of particles per milliliter (#/ml) from the Solids (% solids w/w). The final equation looks like this:

$$\#/\text{mL} = \frac{6\rho_f}{\pi D_p^3 (\rho_f + \rho_b/C - \rho_b)}$$

As a starting point the measured values can be defined as follows:

C Microsphere concentration by mass (%solids/100). This value is determined by weighing out a certain amount of suspension, drying it, and weighing it out again. This number is dimensionless.

ρ_b, ρ_f This is the density of the beads (polystyrene) and the density of the fluid (usually water). Density has units of [grams/cm³]. A typical value for polystyrene is 1.05 g/cm³. A typical value for water is 1.0 g/cm³.

D_p This is the diameter of the particle in centimeters [cm]. The conversion from microns and nanometers to centimeters is:
1 cm = 10⁴µm = 10⁷nm.

The #/mL represents the number of particles per milliliter of suspension, where the suspension is the total volume of particles and fluid.

The total volume of the fluid is the sum of the volume of the beads, V_b , and the volume of the liquid, V_f . Since weights are measured rather than volumes, we can rewrite these volumes as a function of mass and density:

$$V_b = \frac{m_b}{\rho_b} \text{ and } V_f = \frac{m_f}{\rho_f} \quad [\text{mL}] \quad [1,2]$$

where m and ρ are the mass and density of the beads and fluid.

The total volume of the suspension can be expressed as:

$$V_t = \frac{m_b}{\rho_b} + \frac{m_f}{\rho_f} \quad [\text{mL}] \quad [3]$$

Now the denominator is expressed in terms that can be measured.

The numerator represents the total number of particles. This can be written as:

$$\text{Total \# of particles} = \frac{\text{Total mass of particles}}{\text{Mass of a single particle}} = \frac{m_b}{m_1} \quad [\#] \quad [4]$$

The total mass of particles is measured, and the mass of a single particle can be calculated from the mass of a solid sphere:

$$m_1 = \frac{\pi}{6} \rho_b D_p^3 \quad [\text{g}] \quad [5]$$

Particle mass is determined by measuring the percent solids, C . The equation relating mass to the percent solids is:

$$C = \frac{m_b}{m_b + m_f} \quad [\text{dimensionless}] \quad [6]$$

Solving this equation for m_b will put it into a useful form.

$$m_b = \frac{m_f C}{1 - C} \quad [\text{g}] \quad [7]$$

Key Words:

- Particle
- Concentration
- Density
- Diameter
- Equation
- Count

Plugging Equations [5] and [7] into equation [4] results in

$$\# = \frac{m_f C}{\frac{\pi}{6} \rho_b D_p^3 (1 - C)} \quad [\#] \quad [8]$$

Equations [3] and [8] are used to write out the #/mL as a function of known or measured values:

$$\# / \text{mL} = \frac{\frac{m_f C}{1 - C}}{\frac{\pi}{6} \rho_b D_p^3 \left(\frac{m_b}{\rho_b} + \frac{m_f}{\rho_f} \right)} \quad [\#/\text{mL}] \quad [9]$$

Removing both m_b and m_f from this equation will leave just the quantities that can be directly measured. This can be done by substituting equation [7] into equation [9] and simplifying. The end result is:

$$\# / \text{mL} = \frac{\frac{C}{1 - C}}{\frac{\pi}{6} \rho_b D_p^3 \left(\frac{C}{\rho_b} + \frac{1}{\rho_f} \right)} \quad [\#/\text{mL}] \quad [10]$$

The equation can be further simplified by pulling out the concentration, C, terms:

$$\frac{1}{\frac{\pi}{6} \rho_b D_p^3 \left(\frac{1}{\rho_b} + \frac{1 - C}{C \rho_f} \right)} = \frac{1}{\left(\frac{\pi}{6} \rho_b D_p^3 \right) \left(\frac{1}{\rho_b} + \frac{1 - C}{C \rho_f} \right)}$$

Multiplying the two () terms in the denominator results in:

$$\frac{1}{\frac{\pi}{6} D_p^3 + \frac{\pi}{6} \rho_b D_p^3 (1 - C) / C \rho_f}$$

If both terms in the denominator are multiplied by $(C \rho_f / C \rho_f)$, the equation can be simplified a little further, to look like:

$$\frac{C \rho_f}{\frac{\pi}{6} C \rho_f D_p^3 + \frac{\pi}{6} \rho_b D_p^3 (1 - C)}$$

The common terms in the denominator are pulled out and the general equation for the #/mL is:

$$\# / \text{mL} = \frac{C \rho_f}{\frac{\pi}{6} D_p^3 (C \rho_f + \rho_b - \rho_b C)}$$

This can also be written as:

$$\# / \text{mL} = \frac{6 \rho_f}{\pi D_p^3 (\rho_f + \rho_b / C - \rho_b)}$$

Example Calculation:

Polystyrene Microspheres in Water

A bottle of polystyrene microspheres in water has the following properties:

Particle mean diameter:	3.975	microns
Particle density:	1.05	g/cm ³
Particle concentration:	1.0%	solids by weight

Determine the #/mL for this suspension.

Using the equation at the beginning of this paper:

$$\# / \text{mL} = \frac{6 \rho_f}{\pi D_p^3 (\rho_f + \rho_b / C - \rho_b)}$$

We can plug in the appropriate numbers (in the appropriate units) to get the #/mL.

C	Particle Concentration = 1.0%	= 0.01
ρ_b	Density of the beads	= 1.05 g/cm ³
ρ_f	Density of the liquid = Density of water	≈ 1.0 g/cm ³
D_p	Mean diameter of the particles = 3.975 μm	= 3.975 x 10 ⁻⁴ cm
π	The value of Pi	= 3.1416

$$\# / \text{mL} = \frac{(6)(1.0)}{\pi (3.975 \times 10^{-4})^3 (1.0 + 1.05 / 0.01 - 1.05)}$$

$$\# / \text{mL} = \frac{6}{(3.1416) (6.2807 \times 10^{-11}) (104.95)}$$

$$\# / \text{mL} = \frac{6}{(2.0708 \times 10^{-8})}$$

$$\# / \text{mL} = 2.8974 \times 10^8 \approx 2.9 \times 10^8 \# / \text{mL}$$

Since the density of water and the percent solids really only have 2 significant figures, the final result should also only have two significant figures.

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